Problem 6) This problem is similar to that discussed in Sec.6, the only difference being that the radial solution $f(r)$ is now allowed to contain Bessel functions of the 2^{nd} kind in addition to those of the 1^{st} kind. We thus have

$$f(r) = J_m(cr/v) + \alpha Y_m(cr/v), \tag{1}$$

where c is the separation constant, as before, whereas α is a new constant coefficient which specifies the relative contributions of the two Bessel functions in each vibrational mode, m. The boundary conditions now demand that $f(R_1) = 0$ and also $f(R_2) = 0$. Consequently,

$$\alpha = -\frac{J_m(cR_1/v)}{Y_m(cR_1/v)} = -\frac{J_m(cR_2/v)}{Y_m(cR_2/v)}. \tag{2}$$

The separation constant c must ensure the equality of the two J_m/Y_m ratios appearing in the above equation. The acceptable values of c are, therefore, determined numerically, by searching for those values that satisfy the equality of these two ratios. There will be an infinite number of (discrete) values, c_{mn}, that will satisfy the above requirement for each and every mode, m. The corresponding values of α must then be designated α_{mn}, and obtained from Eq.(2). The final solution for the annular membrane will, therefore, have the same general form as that given by Eq.(28), except for $J_m(r_{mn}r/R)$ being replaced by $J_m(c_{mn}r/v) + \alpha_{mn}Y_m(c_{mn}r/v)$. The oscillation frequency of the mn^{th} mode will then be given by $\omega_{mn} = \sqrt{c_{mn}^2 - (\gamma/2)^2}$.
